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ABSTRACT

A mathematics methods course is a critical site for intervening on prospective teachers' past experiences with mathematics, which have often left them without a meaningful understanding of mathematical content, appreciation of what mathematics entails, or confidence in their own ability to do and to learn mathematics. This paper examines the role of a preservice mathematics methods course in helping beginning elementary teacher candidates re-examine and move beyond these past experiences in order to help them learn to teach mathematics for understanding. The paper combines a conceptual analysis of the role of past experiences in learning to teach mathematics with a description of new experiences that were specifically designed to intervene on the past experiences. The new experiences lead the student teachers to reconstruct their prior experiences and to redirect the course of their future with the subject, in this case, mathematics. The conceptual analysis is elaborated with discussion of the prospective teachers' reactions. (Author/JD)

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Deborah Loewenberg Ball



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Issue Paper 89-10

**BREAKING WITH EXPERIENCE IN LEARNING TO TEACH MATHEMATICS:
THE ROLE OF A PRESERVICE METHODS COURSE**

Deborah Loewenberg Ball

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Abstract

A math methods course is a critical site for intervening on prospective teachers' past experiences with mathematics, experiences that have often left them without meaningful understandings of mathematical content, appreciation of what mathematics entails, or confidence in their own ability to do and to learn mathematics. This paper examines the role of a preservice mathematics methods course in helping beginning elementary teacher candidates reexamine and move beyond these past experiences in order to help them learn to teach mathematics for understanding. The paper combines a conceptual analysis of the role of past experiences in learning to teach mathematics with description of new experiences that were specifically designed to intervene on those experiences. The conceptual analysis is elaborated with discussion of the prospective teachers' reactions.

BREAKING WITH EXPERIENCE IN LEARNING TO TEACH MATHEMATICS: THE ROLE OF A PRESERVICE METHODS COURSE¹

Deborah Loewenberg Ball²

This paper examines a particular problem in teacher education: helping prospective elementary teachers learn to teach mathematics. As a mathematics teacher educator, my goal is to help my students learn to do something different from--and better than--what they experienced as pupils in mathematics classes. My problem is also framed by the kinds of experiences with mathematics and with the teaching and learning of mathematics that my students have had before I ever meet them and the ways in which those experiences influence the trajectories on which they move in becoming teachers.

Although teacher educators sometimes speak of preservice teacher education as the first stage in learning to teach, nothing could be further from the truth. In fact, before they take their first professional course, future mathematics teachers have already clocked over 2,000 hours in a specialized "apprenticeship of observation" (Lortie, 1975, p. 61) which has instilled not only traditional images of teaching and learning but has also shaped their understandings of mathematics. My students are also almost all women, as are most prospective elementary teachers, a fact that is significant, given what we know about the widespread alienation of girls from mathematics during their precollege education.

In considering the problem, I focus on the role of a methods course in helping prospective elementary teachers learn to teach mathematics. A methods course is a particular curricular occasion, one that is different from other kinds of teacher education courses in some significant ways. It is about acquiring new ways of thinking about teaching and learning. But it is also about developing pedagogical ways of doing, acting, and being as a teacher. And it is about a particular subject matter--one that brings its own set of issues, different from those in writing or social studies, for instance. What do my students bring to my course and what should I do in trying to influence the direction they move after leaving it, as they develop into teachers of mathematics?

Experience and Learning to Teach Mathematics

I would just use this example (64-46) to present it, and just go through it. I would say, you know, obviously these numbers, you can't subtract in your head. Alright, you have to cross out one of the tens from the top. And put it over in the ones column on the top. So you are able to subtract the two numbers. And then when you cross out that, that tens number, change it, like subtract

¹Paper presented at the annual meeting of the American Educational Research Association, San Francisco, March, 1989.

²Deborah Loewenberg Ball, assistant professor of teacher education, is a senior researcher with the National Center for Research on Teacher Education. She also teaches third-grade mathematics daily in a local public school.

one from it. So you change, like with 64, change it to a, you know, the 6 to a 5, and the 4 to a 14. And then maybe do a few more examples. Go step by step through and show them how to do these. And then have them try a few and if they don't understand it, do it a couple of more times slowly so that they can see exactly what they are doing. Or maybe even take in an example, like use pencils or something, like, maybe take 20 pencils and show them, alright, I have this many, I'm taking away this many, how many does that leave? Just so they have a visual example. And we'd just go from there. They would practice and then when they are comfortable with it I would give them a test. This subtracting is sort of a basic thing, it's not very hard, they either know it or they don't.

Listening to this prospective teacher describe a teaching plan reveals that, although she is just entering a teacher education program, she already has an image of herself teaching math. She has specific ideas about things she could do to introduce students to subtraction of two-digit numbers, including what she would say and how she would explain it. She believes that using concrete materials is a good idea because it gives kids a "visual example." Going slowly step by step is a good idea because they can "see exactly what they are doing." This particular content seems a "basic thing" to her. She will be "presenting"--mostly talking and showing her students how to go step by step. To ensure that they learn, she will give them practice and then, to check on them, she will give them a test. But, she believes, this is straightforward stuff--"either they know it or they don't."

This prospective teacher's image reflects ideas and assumptions about teaching and learning and about pupils that she has picked up outside of any formal professional training. Not surprisingly, these notions are consistent with the way in which mathematics is typically taught in this country. They comprise the "folkways" of teaching mathematics (Buchmann, 1988).

Studies of mathematics teaching suggest that the mathematics classrooms which prospective teachers have experienced have many common denominators. Davis and Hersh (1981) describe the "ordinary math class":

The program is fairly clearcut. We have problems to solve, or a method of calculation to explain, or a theorem to prove. The main work will be done in writing, usually on the blackboard. If the problems are solved, the theorems proved, or the calculations completed, then the teacher and the class know they have completed the daily task. (p. 3)

The teacher (or the textbook) is the authority, theorems are proved by coercion--not reason--and confusions are addressed by repeating the steps in "excruciatingly fine detail" (p. 279). While it makes mathematics educators wring their hands (Kline, 1987), this mode,

elaborated below, represents the dominant approach to mathematics teaching in the United States in which prospective teachers are steeped (Buchmann, 1988).

Stodolsky (1985), drawing from her own and others' findings (Fey, 1978; Goodlad, 1984; Schwille et al., 1983), provides a picture of modal practice: Classrooms are dominated by a recitation and seatwork pattern of textbook-centered instruction. In about 20 of the classes that Stodolsky observed, for instance, students worked individually at their own pace, although most time was spent on whole-group instruction. Rarely did students work in small groups or with partners. Generally, math teachers "introduce new concepts to children and teach and tell them how to do the arithmetic. . . . Once material has been presented to the students, extensive periods of practice are provided" (Stodolsky, 1985, p. 128).

Textbooks dominate this approach to mathematics instruction. Although teachers sometimes omit topics they perceive as "extras," they rarely add mathematical content not covered in the textbook (Schwille et al., 1983). Stodolsky's (1988) analysis of elementary math textbooks suggests that concepts and procedures are often inadequately developed, with just one or two examples given, and an emphasis on "hints and reminders" to students about what to do. She argues that this suggests that it is the teacher's responsibility to develop the ideas in class. Yet, she reports, researchers observe little use of manipulatives or other concrete experiences. Instead, students spend most of their time doing written practice exercises from the textbook.

Known through the common experience of having "been through it," the folkways of school mathematics assume qualities of both obviousness and necessity which "command a moral and cognitive loyalty" (Buchmann, 1988, p. 155). Buchmann argues that, "in learning the folkways, people do not simultaneously internalize the disposition to take a hard look at what they do and what the consequences are" (p. 155). On one hand, teacher candidates who have been successful in mathematics may think that the patterns they have seen are appropriate and therefore may be uninterested in alternative ways of teaching. Those who struggled in math may nevertheless assume that this is the way mathematics *must* be taught and that they are simply among the "have-nots" in mathematics. On the other hand, they may also aspire to teach differently. But even if prospective teachers are critical of their own past teachers for teaching badly and for making them feel stupid, many of them lack alternative images of mathematics teaching, having had no other models.

Furthermore, and equally serious, what we know about what students learn in ordinary mathematics classes suggests that prospective teachers are unlikely to know math in the ways that they will need to in order to teach. Their years in school have also shaped their understandings of mathematics. As this is the mathematics they will *teach*, what they have learned about the subject matter in elementary and high school turns out to be a significant component of their preparation for teaching (Ball, in press; Ball and McDiarmid, in press). For example, interviews with prospective elementary teachers revealed that few

of them understood the conceptual principles underlying the procedure for subtraction of multidigit numbers (i.e., "borrowing") [Ball, 1988]. "I don't really understand the technical, you know, *why's* that you do it," explained one. And almost never have prospective teachers had any opportunity to engage in mathematical discourse--make conjectures, to construct arguments, to challenge others' claims--or to develop a sense of what it might mean to work on mathematics within a mathematical community.

Finally, the picture painted above is even more dramatic in the case of women, whose experience with mathematics in school tends to be still more alienating than that of students in general. And prospective elementary teachers are predominantly female. This cannot be overlooked and it presents the mathematics teacher educator with yet another problem, a problem related to the special nature of the population with whom we work. Unless mathematics teacher educators are satisfied with what prospective teachers have learned from their experiences as students in math classrooms (and most are not), this highlights a need to interrupt, to break in, what is otherwise a smooth continuity from student to teacher. In her paper, Margret Buchmann (1989) asks, "Are breaks with experience necessary in teacher education?" In the case of helping people learn to teach mathematics, my answer is yes, but also no.

What Does It Mean to Break With Experience?

Past experience necessarily affects the present. As Dewey (1938) writes, "every experience both takes up something from those which have gone before and modifies in some way the quality of those which come after" (p. 35). What we have seen, thought, and felt affects our immediate perceptions, interpretations, and habits. We think that it will rain because we have seen dark, humid skies like this before. We walk cautiously on ice because we have slipped before. Prospective teachers can behave like teachers; they can identify and correct children's subtraction papers, tell who is paying attention, and assign homework--all based on past experience as students in classrooms. Experience allows us to develop routines and frames of reference that simplify life; some things become automatic, less puzzling, easier.

Yet herein lies the dilemma. What is learned from experience is not subject to scrutiny, to appraisals of worth or defensibility; all conclusions are not equally desirable. As Dewey (1938) notes, "experience and education cannot be directly equated to each other. For some experiences are miseducative. Any experience is miseducative that has the effect of arresting or distorting the growth of further experience" (p. 26). Experiences may inhibit openmindedness, freeze ways of looking, or engender undesirable attitudes. Experiences can therefore limit our possibilities for continued learning. In the case of prospective mathematics teachers, their experiences have often persuaded them that mathematics is a fixed body of rules, a dull and uninteresting subject best taught through memorization and

drill, and that they themselves are not good at math. They have developed the idea that math teaching involves giving directions about what to do, assigning work, and, as one of my students wrote, "sit at the desk and wait for people to come up for extra help or to get their papers checked." Consequently, prospective teachers, equipped with vivid images to guide their actions, are inclined to teach just as they were taught. Given the widespread criticism of mathematics education in this country (e.g., Dossey, Mullis, Lindquist, and Chambers, 1987; National Research Council, 1989; Paulos, 1988) this consequence is of substantial concern.

If the principle of continuity of experience is inevitable, what does that imply for the educator, one who wishes to shape and affect others' futures? The responsibilities are twofold. First, educators must judge what prior learnings can contribute to future growth and which may impede it. This implies a need to examine what learners bring--what they already know, believe, assume, and are inclined to do. Educators must also have a vision of where learners are headed and what ideas, beliefs, attitudes, and dispositions are likely to prove useful for moving in that direction. Second, educators must be able to construct the conditions for experiences which can foster future growth:

As an individual passes from one situation to another, his world, his environment expands or contracts. He does not find himself living in a different world but in a different part or a different aspect of one and the same world. (Dewey, 1938, p. 44)

The educator's goal, therefore, is to intervene in the inevitable continuity of experience in ways that affect its future quality and direction. This involves a kind of conceptual change, perhaps as Petrie (1981) conceives it. He argues that conceptual change--instances when individuals come to think or see differently--may involve one or more of the following: changes in meaning, changes in perception, changes in methodology (p. 46). Interestingly, most discussions of individual conceptual change treat it as discontinuous, as a radical departure from prior ways of thinking.³ In fact, conceptual change may be seen as part of the continuity of growth. On one hand, future experiences are affected, redirected, by such changes in ideas, ways of seeing, or ways of doing things. On the other, however, past experiences can also be reinterpreted and reconstructed, given new lenses, new assumptions, new ideas.

³This may derive from its origins in the history of science. Although conceptual change is most often used as a theoretical perspective for individual learning and cognition, its roots are in the revolutionary paradigm shifts described by Kuhn (1970) and other philosophers of science. As Buchmann (1989) points out, based on Kuhn, "after changing their world view, scientists work in a different world" (p. 22). Prior scientific theories are dismissed as wrong or are forgotten. This characteristic of change in a discipline may not apply well to changes in individuals.

It is this notion of interrupting the continuity of experience with mathematics and the teaching and learning of mathematics that underlies the work I have been doing with prospective elementary teachers in a preservice methods course. Does this represent a break with experience? Yes, in the sense that I intervene. But no, given that my aim is to help my students reinterpret their past experiences with mathematics and to redirect their future experiences with it. In this latter sense, I respect the continuity of their learning, both as students and as teachers of mathematics. It is that past experience, however reinterpreted, that necessarily provides the support and impetus for future learning. For example, that the prospective teachers have themselves had such frustrating experiences with mathematics can serve to fill them with desire to provide their own students with a better alternative.

What is a Methods Course?

A methods course faces a tension not faced by other courses: a tension that reflects the fundamental nature of teaching. Teaching is about weaving together knowledge about subject matter with knowledge about children and how they learn, about the teacher's role, and about classroom life and its role in student learning. An educational psychology course can focus on theories of learning. A mathematics course can be about algebra, or geometry, or combinatorics. But a methods course can be about the weaving that produces teaching. As such, a math methods course is about mathematics. It is also about children as learners of mathematics, about how mathematics can be learned--and taught, about how classrooms can be environments for learning math. The complexity of teaching coupled with a sense of the continuum of learning to teach (Feiman-Nemser, 1983) makes a methods course perhaps an impossible concept. I return to my original question: What can a math methods course try to *do*?

Methods courses are the mainstay of traditional teacher education programs. Prospective teachers typically look forward to them because they expect that they will learn how to teach specific things. These are the "practical" classes, unlike foundations or general education courses. On the first day of class, one of my students wrote that she hoped "to get some creative ideas about how to teach math and ideas about how to help kids learn their facts." Another said that "math is difficult to explain--this is something I want to work on this term." Many hoped to learn how to get better at "getting concepts across" to students, how to explain skills more clearly, and how to make math more fun and relevant.

My students' expectations seem both too high and too low to me. On one hand, they hope they can learn how to teach math from this course, a course which lasts for only 10 weeks, meeting about four hours a week. These are high expectations. On the other hand, my students do not expect the course to challenge what they already know about teaching

mathematics. They want to get better at what they know math teachers have to do: explain, show, and tell. From my perspective, these expectations are low.

I have often thought of what I do in the space of 10 weeks as "launching." I have thought that I am trying to "launch" them as learners of teaching, to equip them with ideas, ways of thinking, commitments, and ways of acting that will serve them well in continuing to learn on their own, from their own experience as teachers of mathematics.⁴ Different from a foundations course, a methods course *is* about more than ideas. It is about developing ways of acting as well as ways of thinking. For example, my students come with the habit of asking children about their answers to math problems only when the answer is wrong. "Is $6 + 8$ equal to 12?" they are inclined to ask, in a tone and with an expression that makes clear that $6 + 8$ is most definitely *not* 12. They are not in the habit of asking, "And *how* did you come up with 36?" when 36 is right. Instead, they approve the correct answers without further discussion ("Good!") and query the incorrect ones.

When a child asks, "Is this one right?" my students are inclined to check the child's answer and tell her if it's correct or not. They are not accustomed to returning the question, for instance: "Can you show that it makes sense?" When a child asks for help with a problem, they tell him how to do it. They are not in the habit of encouraging him to confer with other children about the problem. A methods course is, in part, about ways of acting, ways of doing--about methodology (Petrie, 1981).

Methods courses are of course also about ideas and about ways of seeing. When watching children struggle with a difficult problem, many of my students infer that the children are frustrated--and, therefore, uncomfortable and unhappy. When they see children disagree and argue about a solution to a problem, they think that the children are confused, that the teacher should step in and explain. When they see a child revise a solution in front of the class because of something that another child pointed out about his approach, many of them assume that he is embarrassed about having been wrong. These interpretations, these ways of seeing, are often projections of how they would feel in similar situations.

Prospective teachers make other assumptions as well: When children help each other with problems, they are not really learning the material. If students use manipulatives or draw a picture to solve a problem, they don't yet fully understand it. Word problems are what is meant by problems. Kids can't solve problems which they haven't been shown how to do. Learning to reconsider these conclusions and to understand their sources is part of what a methods course must be about. Learning alternative frames of reference is another.

⁴This metaphor, however, is flawed, for it suggests that this is the *beginning*. In fact, as I have argued above, my methods course comes in the middle, following years of constructing assumptions about mathematics and what it means to know and to do mathematics, about themselves in relation to mathematics, about the roles of teachers and students, and about classrooms as contexts for learning.

Learning to Teach Mathematics: Breaking the Continuity

In designing a mathematics methods course, I have chosen two broad areas in which to intervene: (a) prospective teachers' knowledge, assumptions, and feelings about mathematics and about themselves in relation to mathematics and (b) their assumptions about classrooms and the roles of teachers and students in learning mathematics--and their associated ways of acting based on these assumptions. These obviously overlap, for as we work on mathematics together as a class, we are also operating within a classroom community that represents notions about learning and teaching that I want them to notice and consider. The interplay between these is reminiscent of Schwab's (1971) point about the two meanings of "learning community": One may *learn* about the notion of community, and one may learn within a *learning community*. Similarly, prospective teachers may learn about learning mathematics while they are themselves learning mathematics.

We work within two learning communities: the methods class itself and the public school third-grade mathematics class in which I teach daily and in which the prospective teachers participate intermittently throughout the term. Using the school classroom affords an instructional opportunity whose nature is uniquely between field experience and videotape. Its texture of reality and interactive nature embody the power of field experience. The prospective teachers can work with children, ask them questions, try things out and examine their consequences. For example, they interviewed the children to learn more about what they were learning as well as to learn what it's like to try to find out what pupils are thinking. That the experience is both shared and controlled resembles videotape. We can discuss what happened in the class because we were all there. And, despite the obvious uncertainties of classroom teaching, I have a great deal of control over what the prospective teachers encounter in my classroom. This is not the case with their regular field experiences, many of which represent more of the same mathematics teaching and learning on which the prospective teachers were raised and, although good sites for experimenting with mathematical pedagogy, are not well suited to interrupting the continuity of their experience.

In certain ways, the methods classroom and the third-grade classroom mirror each other. My role in each looks very similar: I pose tasks, encourage people to collaborate, to generate solutions and supporting justifications. I orchestrate group discussion of the problems and their solutions, encouraging students to participate by questioning, challenging, corroborating others' ideas.⁵ The third-grade class gives the prospective teachers a vision of what this might look like in a regular classroom, a sense of possibility. As a mirror, it helps to focus attention on features of our classroom community--what I do in my role as teacher, for example. It also helps them to learn to learn mathematics in some new ways. They

⁵See Ball (1989) for an extended narrative example of this in the third-grade class.

giggle when they first notice that they have picked up ways of talking from the little kids, saying, for instance, "I'd like to challenge what Jenny said."

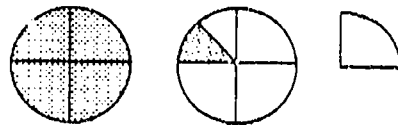
The third-grade class does not, however, just mirror the methods class. It also contrasts with it, for the third graders do not bring the same baggage to the learning experience as do the prospective teachers. For example, the third graders depend on me less and on one another more. They do not construe that as cheating but as a sensible way to work. They employ and invent a wide variety of strategies for solving problems: in their innocence, they are not bound to find "the formula." They have considerably more confidence in their solutions and speak with noticeably more authority. They challenge one another freely, revise their ideas without apparent embarrassment, and use sophisticated mathematical language and ideas (e.g., conjectures, negative numbers). The third graders provide a challenge to prospective teachers' assumptions about what young children can do and understand, about what school learning of mathematics must entail (e.g., lots of practice, a quiet environment, sugar-coating to make the mathematics interesting or fun).

This double experience with learning mathematics provokes some of the prospective teachers to reinterpret their own past experience with mathematics. It suggests to them, for the first time, that the reason they feel anxious about and incompetent with mathematics is *not* due to some shortcoming on their part. That they feel the way they do, that they lack the understandings they do, may instead be the product of the math classrooms in which they were students. This is an encouraging new interpretation, one that seems to inspire some renewed confidence as well as some disposition to learn (or relearn) mathematics. Many students comment on this. One student wrote, "I have a brand new outlook. . . . This class has totally changed my math thinking from my past 22 years of learning it technically the wrong way. . . . I have established a confidence in myself." And another wished she could now "go back and really understand" the math she had been taught in high school.

While the experience gives prospective teachers a vision and provokes them to reexamine their own past experiences, it is also unsettling. As long as the image of mathematics teaching looks much like what they remember from school, they can imagine being able to do it. This alternative, and the accompanying evidence that eight-year-olds can understand negative numbers, come up with insightful conjectures, and invent novel strategies is a bit scary. This tension between instilling new confidence and unsettling old assumptions is perhaps inherent in breaking the continuity of prospective teachers' learning. On one hand, their reinterpretation of their past gives them new encouragement. On the other, the alternative vision suggests that teaching mathematics, even to young children, may be less straightforward than it looked before. I will return to this tension below.

A course about the teaching and learning of mathematics is necessarily about mathematics: particular content as well as epistemological issues. Once again, I want to provide experiences that lead them to reconstruct what has come before as well as to

redirect the course of their future with the subject. One day, for instance, I asked the prospective teachers to calculate $2\frac{1}{4} \div \frac{1}{2}$. Although some felt anxious and said they were "rusty," they were all able to remember the principle of "invert and multiply" and to get the answer of $4\frac{1}{2}$. Then I asked them to write some kind of story that would go with $2\frac{1}{4} \div \frac{1}{2}$. Several wrote stories such as the following: "I had $2\frac{1}{4}$ pizzas and I gave half to my friend. We each got $4\frac{1}{2}$ pieces of pizza." Their pictures looked like this, showing that they divided the pizza into nine *fourths* and then divided those nine fourths *in half* to get $4\frac{1}{2}$ fourths or $1\frac{1}{8}$:



Others struggled with the task and, working together, could not produce a story that seemed to go with $2\frac{1}{4} \div \frac{1}{2}$. The $4\frac{1}{2}$ just did not seem to fit. They began to wonder if their original answer had been right. Maybe it should be $1\frac{1}{8}$. As they discussed the problem and examined different possible solutions, several began to understand that the question was about how many halves there were in $2\frac{1}{4}$. They tried to explain this to others. Slowly, flickers of understanding grew until the group reached a shared sense of what it *meant* to divide $2\frac{1}{4}$ by $\frac{1}{2}$. Many of them wrote about this experience later. For example, one wrote:

I realize now that I didn't really understand many of the manipulations that I could produce the correct response for. Working with the [fraction problem] was a real eye opener for me. While I could quickly come up with a correct answer to the problem, I had no idea how to write a story for it. Finally, through discussion with others and my own thinking out loud, I realized what the problem was asking me to do. After 16 plus years of school I understood division of fractions for the first time.

Experiences like this can serve to help prospective teachers revisit and reconstruct mathematical ideas they have "learned" before. Through this and other experiences, these students had an opportunity to think about and understand division in a way they never had before. They were able to talk about what division *means*, and to write stories that illustrated different meanings of division in different contexts, and to integrate, for the first time, division of fractions with division of whole numbers.

Experiences like this can also help prospective teachers acquire a new sense of what it means to "understand" something in mathematics: That understanding does not mean simply knowing "how to do it." Several students identified the fraction problem as one of the two or three key experiences of the term, not for what they learned about division of fractions, but for what they learned about what it means to *understand* something in mathematics. For example:

I thought of math as rigid. There was always only one answer and you got it the easiest way possible, usually by an algorithm. Now I see math as much more flexible as a subject. . . . I think what mainly changed it was the lesson we did on fractions. . . . I felt extra good when I did understand it. . . . It seems to me that I know more about math as a subject than I did before.

Two class quizzes offered another opportunity to consider what mathematical knowing entails. Students were asked to work both alone and with others to solve problems and to write explanations to justify their solutions. One student struggled with articulating a justification for her manipulations in a standard subtraction problem:

I can easily calculate the problem

$$\begin{array}{r} 261 \\ -145 \\ \hline \end{array}$$

However when I had to explain how I *understood* the problem, I could not. I was merely a victim of rules and procedures. I could not explain the problem in terms of tens and ones. When I finally figured it out, it seemed so easy to me.

Another wrote:

When I walked into this class this fall, mathematics to me was knowing how to get the right answer--not so much even HOW to get it as merely GETTING it!! I felt that the best way to know math was to have a photogenic memory. . . . My greatest nuisance was story problems. I grew up hating them. Now my ideas are completely different. Mathematics is not memorization, it's more like methodical reasoning. Now one of my favorite kinds of problems is story problems because I can take it apart and figure out what it is asking for. Before I would take on the whole problem at once and *hope* I got it right. I was never able to look at it and say, "Yes! For sure that is right because . . . " Now I know WHY it's correct or if it's not.

A third experience that offered opportunities for reconsidering what knowing mathematics entails was observing and working with the third graders. One student said that it made her feel "really weird" seeing the children discussing mathematical ideas; she had never thought of mathematical ideas as discussable. After all, as many remarked, "if math is a simple matter of right or wrong, what's to discuss?" Gradually they began to see subtleties in the content, significant issues worth debating. The third graders were embroiled for several days in a discussion of whether zero is even, odd, or neither. Children, individually and in small groups, marshalled arguments in attempts to persuade others of

their views. Other issues which the third graders were working out and in which the prospective teachers became engaged included what it means to subtract a negative number and the definition of even numbers. The children's work on these issues, the class discussions, the efforts to achieve community consensus pushed the prospective teachers to reconsider their assumptions about mathematical knowledge and ways of knowing. One of my greatest pleasures of the term was finding a small group of them sitting in the lobby of the College of Education arguing about whether or not 1 was a prime number.

Having a different notion of what "knowing" entails may make a difference in what prospective teachers try to learn as well as what they strive for with their own students. In this way, the experience of learning mathematics can serve to break the continuity of their experience with the subject in ways that have the potential to affect both their past and future.

Again, however, there is a tension here. Although such experiences may have effects that enhance prospective teachers' capacity to grow, they may also discourage: After all, if this is what is entailed in understanding mathematics, then I really know even less than I thought I did. And where will I ever learn all the things I need to understand if I want my own students to understand the mathematics they are learning?

Problems in Attempting to Break the Continuity

Preservice teacher education is fraught with tensions and dilemmas. Two stand out in trying to interrupt the continuity of prospective teachers' learning to teach mathematics: one related to prospective teachers' thin understanding of mathematics, the other related to time.

Prospective teachers' thin understanding of subject matter produces one major tension. Years of memorization, of focusing on answers, of inattention to meanings, have yielded reliably algorithmic ways of knowing and doing mathematics. Furthermore, the surrounding culture is no less oriented toward mathematical sense-making. Certainly in 10 weeks we can revisit and unpack a very few--although, if carefully chosen, powerful--ideas means that prospective teachers are left to reinterpret and learn a lot of mathematics. Can they take additional courses that will help them to do more of this? Not likely at most universities (Kline, 1977). And, if the continuity of learning to teach mathematics involves an interplay of learning mathematics themselves with learning about learning, then the limits of what prospective teachers understand may prove a substantial obstacle to learning to teach *mathematics* for understanding.

Ten weeks, four hours a week, is a minuscule, almost trivial, amount of time to contemplate the agenda I have set. The risk is that the tension between unsettling assumptions and generating future growth will be left unbalanced and that the continuity will be therefore uninterrupted. Prospective teachers may come away even less confident

than they were before, more worried that they will not be able to teach mathematics so that kids can understand. They may see classrooms and children as daunting, mathematics as a vast sea of things they really do not understand. Thus unsettled, the most logical course of action would be to return to the safety of the old assumptions and habits. They are comfortable and familiar. It is risky business to foster the kind of conceptual change that Petrie (1981) describes, for it entails changes in meanings, ways of seeing, and ways of acting, but within a very familiar world. The old world view is comfortingly just around the corner should the new one prove inadequate. Continuity is easily restored, the trajectory of future growth unaffected.

Furthermore, in a short period of time, only some things change. For instance, many of my students became persuaded that representation was a key part of learning to understand mathematics. They may have, however, had time only to develop the idea that pictures, stories, concrete materials, and the like are helpful. They may have developed a dogmatic view of "manipulatives." They are probably not prepared to learn from practice the relative merits of alternative representations. When are bundling sticks a better choice than base 10 blocks? Or does it matter?

At the end of the term there is evidence that the experience has had an impact on the prospective teachers' ideas, ways of seeing, and ways of acting. The extent to which this impact in fact can help to redirect the continuity of their learning to teach mathematics is an empirical question, well worth asking. But, skeptical in any case of the adequacy of a 10-week course, I think it equally worth pursuing how one might extend its duration and form in ways that would make it more likely that we could prepare teachers to learn from their own practice.

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